SOME RECENT GRAPHICAL CONTROLS FOR PROCESS INSPECTION

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1. The principal aim of process inspection is to furnish us information, either to assure us that the process is producing its output according to specifications, or to warn us promptly that some departure from specifications is occuring so that action may be taken. Usually a graphical presentation of the gathered information enables us to readily identify the occurrences of trouble in the process and to make easier isolation of the causes of trouble. Similarly, the graph easily draws our attention to any significant improvements in the process and to isolate the cause of such improvements.

Graphical controls are very helpful devices if they significantly indicate from a large mass of data when action or decision is necessary. In this paper some of the recent graphical controls for continuous process inspection are presented. The discussion begins with the construction and operation of the standard control chart owing to W. A. Shewhart.

2. The most common and the longest established statistical form of graphical control for continuous process is due to W. A. Shewhart (1931). Since his original proposals, this control chart has undergone various modifications and amendments.

In setting up a Shewhart control chart for any statistics θ, the central line (CL) will be drawn at μ_θ with the upper control limit (UCL) and the lower control limit (LCL) at distances $+3\sigma_\theta$ and $-3\sigma_\theta$ respectively, from CL where $\mu_\theta = E(\theta)$ and $\sigma_\theta^2 = V(\theta)$.

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The values of μ_θ and σ_θ may be given either in the form of specified values or in the form of estimates based upon past data. In case both are not available a suitable estimate may be obtained from a random sample.

The operation usually consists of plotting points obtained from samples on the control chart. If any point is observed to be outside the control limits then lack of control is indicated.

For illustration, consider a set of hypothetical measurement data arranged in 20 groups of 5 observations each taken from a production process available (see Table 1). The problem is to set up a control chart. An estimate of the process mean is obtained by taking the mean of the 20 means. The estimated mean is $\bar{\bar{x}} = \frac{1}{20}\sum_{i=1}^{20} \bar{x}_i = 0.8312$ and the CL passes through this value on the chart. An estimate of the process standard deviation is similarly estimated from the mean of 20 sample ranges. Thus, the mean of the ranges is $\bar{R} = \frac{1}{20}\sum_{i=1}^{20} R_i = 0.01435$ Therefore, the control limits for the process mean are

$$UCL = \bar{\bar{x}} + A_2\bar{R} = 0.8312 + 0.577\ (0.01435) = 0.8395$$

$$LCL = \bar{\bar{x}} - A_2\bar{R} = 0.8312 - 0.577\ (0.01435) = 0.8229$$

where A_2 are constants which can be obtained from prepared tables.

Let us suppose that we have the 10 observations shown in Table 1 as sample numbers 21 to 30 and we want to find out whether there is lack of control on the process average. The control chart is shown in Figure 1. The 29th and 30th points fall outside the UCL indicating lack of control of the process average.

A control chart for the process standard deviation may be constructed in a similar manner and the 10 sample ranges found in Table 1 may be plotted as shown in Figure 2 where

$$UCL = D_4 R = 2.115\ (0.1435) = 0.03036$$

where D_4 are constants which are available from tables.

TABLE 1

$\bar{X}$ AND R FOR SAMPLES OF FIVE MEASUREMENTS

Sample Number	$\bar{x}$	R
1	0.8324	0.014
2	0.8306	0.008
3	0.8262	0.020
4	0.8326	0.004
5	0.8290	0.013
6	0.8316	0.013
7	0.8336	0.012
8	0.8310	0.020
9	0.8336	0.010
10	0.8306	0.010
11	0.8332	0.018
12	0.8288	0.006
13	0.8310	0.016
14	0.8294	0.023
15	0.8322	0.003
16	0.8288	0.025
17	0.8344	0.016
18	0.8270	0.023
19	0.8338	0.025
20	0.8332	0.007
	$\bar{\bar{X}}$ 0.8312	$\bar{R} = 0.01435$
21	0.8310	0.012
22	0.8304	0.009
23	0.8328	0.005
24	0.8332	0.016
25	0.8282	0.022

26	0.8306	0.028
27	0.8354	0.030
28	0.8385	0.023
29	0.8410	0.031
30	0.08415	0.032

3. Various rules about runs of points have been suggested to overcome disadvantages of taking action based on one or, at most, two sample points on a control chart. Attempts to run together the information from several successive sample points have resulted in charts utilizing information from the past sample points. In this case, a very useful chart is the arithmetic running mean (ARM) chart (Roberts, 1959).

The construction of the CL of an ARM control chart is the same as in the Shewhart control chart except for the UCL and LCL which will now be drawn at distances $+\frac{3}{\sqrt{k}}\sigma_\theta$ and $-\frac{3}{\sqrt{k}}\sigma_\theta$ respectively from CL where k is the number of running points to get a point on the ARM chart. Suppose the first k running points are $\bar{x}_1, \bar{x}_2 \ldots \bar{x}_k$, then the first point on the ARM chart is

$$y_1 = \frac{1}{k} \sum_{j=1}^{k} \bar{x}_j .$$

If a new point $\bar{x}_{k+1}$ is available, then the next point will be

$$y_2 = \frac{1}{k} \sum_{j=1}^{k} \bar{x}_{j+1};$$

and so on. Thus, the points plotted on the ARM control chart are the means of k running points or the moving means $y_1, y_2, y_3, \ldots$.

Lack of control is indicated in this chart by the running mean falling above (or below) a single control limit. It is, however, necessary to take a prior decision on k, the number of sample points on which the running mean is to be based.

For illustration, let us consider the same problem as in the last section but using this time ARM control chart. The pertinent data for constructing the chart in the case where k = 4 are the following:

$$CL = \bar{\bar{x}} = 0.8312$$

$$UCL = \bar{\bar{x}} + A_2\bar{R}/\sqrt{k} = 0.8353$$

$$LCL = \bar{\bar{x}} - A_2\bar{R}/\sqrt{k} = 0.8271$$

The running mean computed on the basis of k = 4 for the sample numbers 21 to 30 are shown below. These

Sample Number	$\bar{x}$	y
21	0.8310	—
22	0.8304	—
23	0.8328	—
24	0.8332	0.8319
25	0.8282	0.8311
26	0.8306	0.8312
27	0.8354	0.8319
28	0.8385	0.8332
29	0.8410	0.8364
30	0.8415	0.8391

are plotted in the ARM control chart shown in Figure 3.

4. It will be observed in the ARM chart that it utilizes a special form of weighted mean of past results. One form of chart in which the weights to be used get progressively smaller as the results become more distant in time is the exponentially weighted mean (EWM) chart (Roberts, 1959). The weights change progressively by the factor (1-W) where $0 < W < 1$. In constructing the chart, it is necessary to decide in advance the value of W by which the successive weights decrease. When computing the EWM, the most recent mean is assigned a weight of W and every other mean is assigned a weight (1-W) of the weight of its immediate successor.

Suppose $x_1, x_2, x_3, \ldots, x_i, \ldots$ are sequences of observed sample means taken from a continuous process. Let z_i be the EWM for the i^{th} point and let W be the chosen weight. Then we have

$$z_i = (1-W)\, z_{i-1} + W\, \bar{x}_i$$

and z_0 will be taken as the target value so that

$$z_0 = CL = \bar{\bar{x}}.$$

Thus, the point z_i is located a fraction W of the distance from z_{i-1} to $\bar{x}_i$ on their connecting straight line. At the limit as i increases the standard deviation of z is given by

$$\sigma_z = \qquad \sigma_{\bar{x}}.$$

For illustration in constructing an EWM control chart, let us use the same data given in section 2 and let $W = 2/5$ which was shown to have wide appeal in practice. Then the necessary data for the control limits are

$$CL = z_o = 0.8312$$

$$UCL = z_o + \sqrt{W/(2-W)}\ A_2\bar{R} = 0.83535$$

$$LCL \quad z_o - \sqrt{W/(2-W)}\ A_2\bar{R} = 0.82705$$

Suppose, we plot the EWM of the sample means for the sample numbers 21 to 30 from Table 1. The computed EWM are shown in the table below:

Sample Number	x	z
21	0.8310	0.8311
22	0.8304	0.8308
23	0.8328	0.8316
24	0.8332	0.8322
25	0.8282	0.8306

26	0.8306	0.8306
27	0.8354	0.8325
28	0.8385	0.8349
29	0.8410	0.8373
30	0.8415	0.8390

The plotted points are shown in the EWM chart in Figure 4. Lack of control is indicated by an EWM falling above the UCL.

5. A chart which provides a means of presenting visually at any instant, the apparent mean of any group of sample observations is the cumulative sum (cu-sum) control chart which is due to (Page, 1954, 1961). This chart is intended to replace the standard form of control chart. In a cu-sum control chart, cumulative totals are plotted against the number of sample observations.

Suppose $\bar{x}_i$ denote the mean of the i^{th} sample and $\sigma_{\bar{x}}$ the known standard deviation of $\bar{x}_i$. Then it is convenient to consider the points on the cu-sum chart as having coordinates (m, S_m) where

$$S_m = \sum_{i=1}^{m} \frac{(\bar{x}_i - \mu)}{\sigma_{\bar{x}}}$$

and μ is the target value. However, in practice it would be sufficient to calculate

$$\sum_{i=1}^{m} (\bar{x}_i - \mu)$$

and then use an appropriate scale factor.

The cu-sum chart is interpreted by placing a V-mask (Bornard, 1959) over the chart, with the vertex of the V placed a distance d from the last plotted point on the chart and the axis of symmetry of the V held horizontal. If any points of the cu-sum chart are covered by the mask or if any points lie below the lower limb or above the upper limb of the V, then lack of control is indicated. The control limits are the limbs of the V mask. The dimensions of the mask are defined by

the distance d and the angle θ . a limb makes with the axis of symmetry of the letter V. Johnson (1961) using a sequential technique suggested a simple formula for determining d and θ which are dependent only on the desired size of absolute change in the target value D and the greatest tolerable probability, $2\alpha_o$ of false indication of lack of control. These are

$$\theta = \tan^{-1}(\tfrac{1}{2}\delta)$$

and

$$d = -2\delta^{-2}\log_e \alpha_o,$$

where

$$\delta = D/\sigma_{\bar{x}}.$$

The cu-sum chart is in effect a running mean chart and found in practice to be more effective in detecting sustained changes than the standard control chart. On the other hand, the standard control chart is more effective in detecting larger, shorter term changes and is extremely simple to apply.

For illustration, let us again consider the sample means of sample numbers 21 to 30 of Table 2 and with the given values: $\mu = 0.8300$, $\sigma = 0.02237$. Suppose that the desired absolute change from the target value is 0.0010 with a probability of wrong decision as 0.10. Then $\delta = 1.000$, $\theta = \tan^{-1}(0.5) = 26°30^1$ and d = 6. Thus with the computed values of θ and d the control limits of the cu-sum chart is fixed.

The cu-sum points corresponding to the sample means of sample numbers 21 to 30 are shown in the following table:

Sample Number	$\bar{x}$	S
21	0.8310	1.0
22	0.8304	1.4
23	0.8328	4.2
24	0.8332	7.4
25	0.8282	5.6
26	0.8306	6.2
27	0.8354	11.6
28	0.8385	20.1
29	0.8418	31.1
30	0.8418	42.6

Plotting the points on the cu-sum control chart and using a V mask defined by $\theta = 26^\circ 30^1$ and d = 6, we note that at the 28th point there are points covered by the mask indicating lack of control.

REFERENCES

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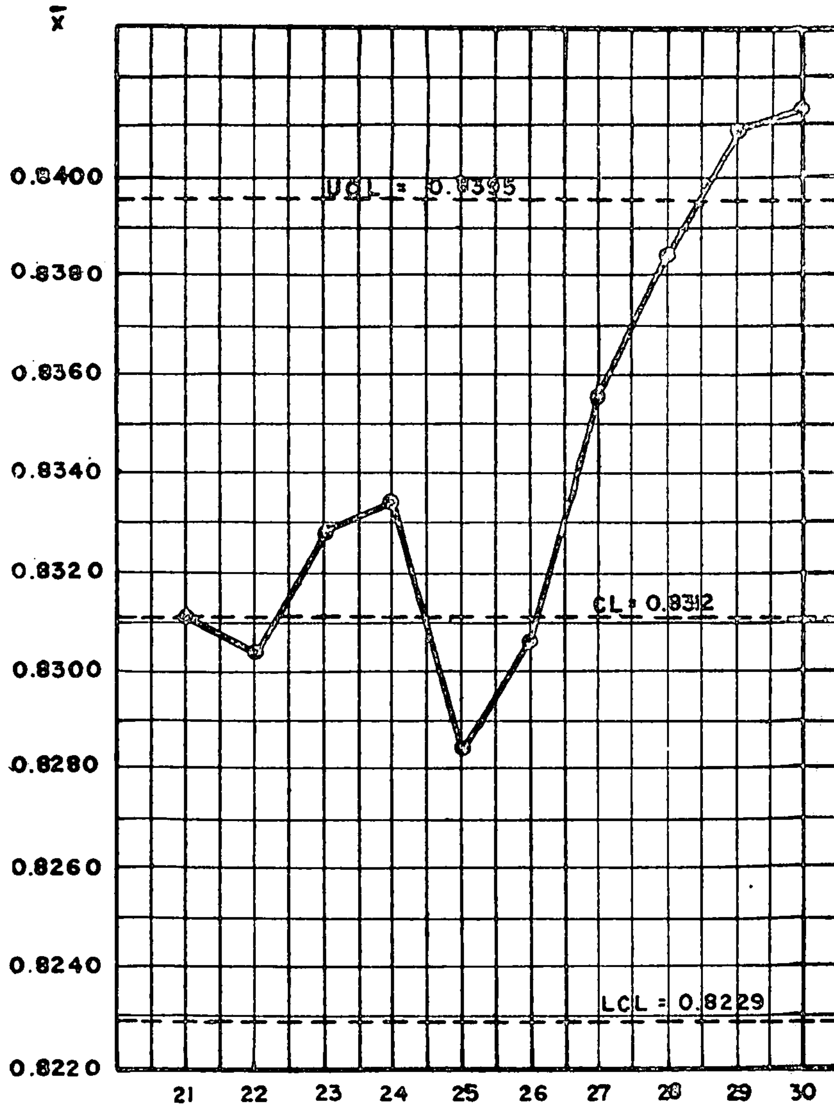
FIGURE 1. AN $\bar{X}$-CHART IN OPERATION



SAMPLE NUMBER

FIGURE 2. AN R-CHART IN OPERATION

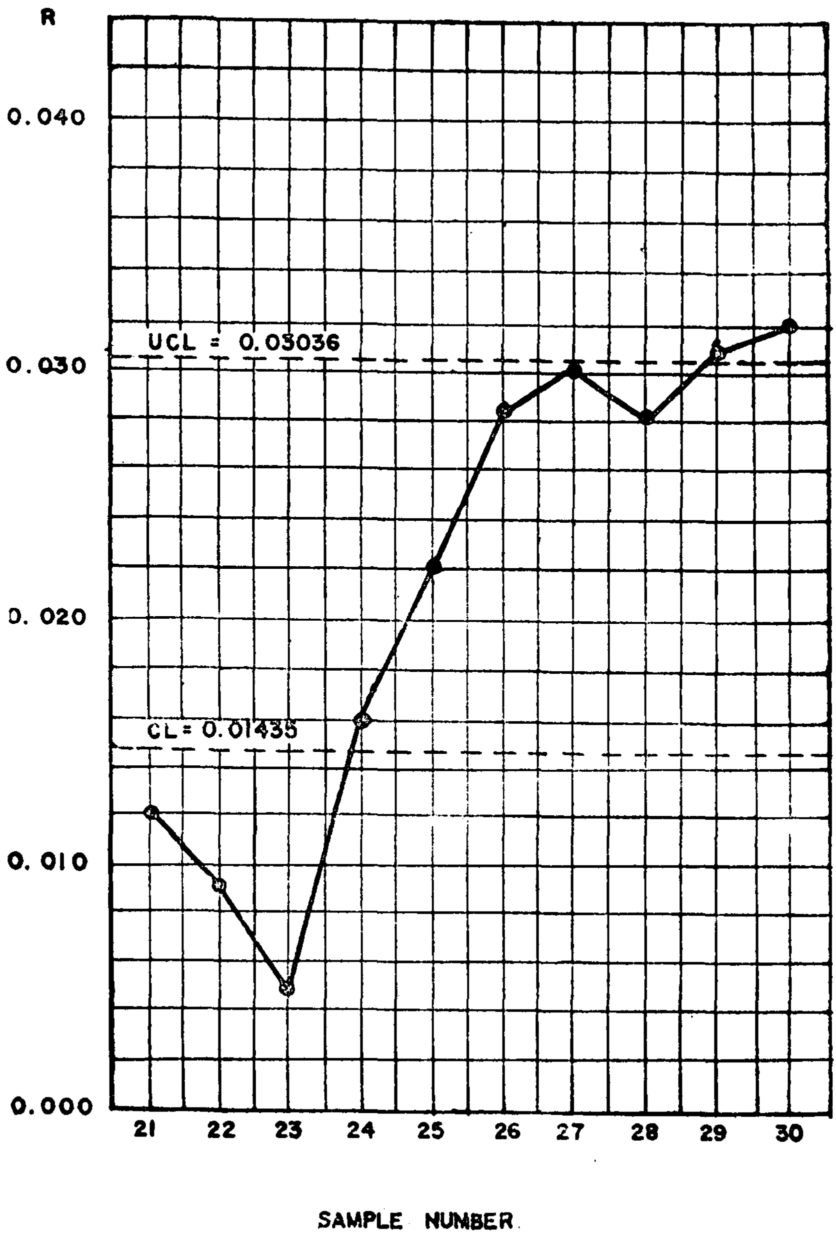


FIGURE 3. AN ARM CHART IN OPERATION

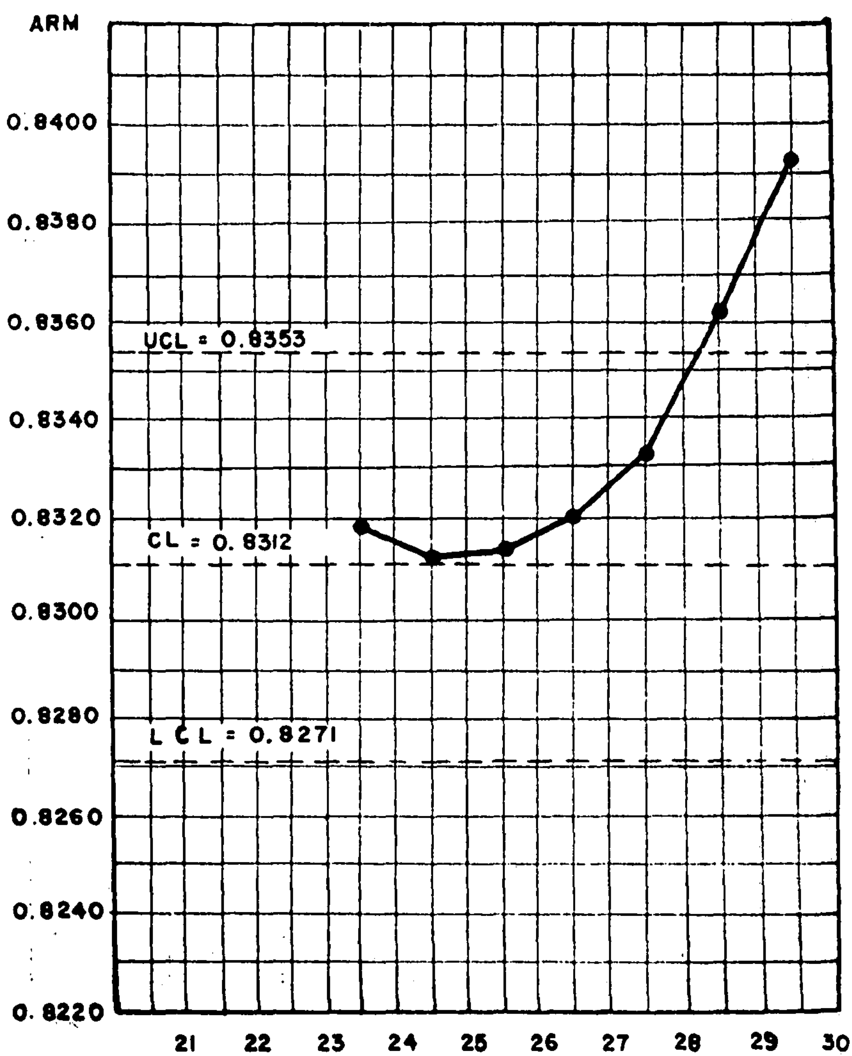


FIGURE 4. AN EWM CHART IN OPERATION

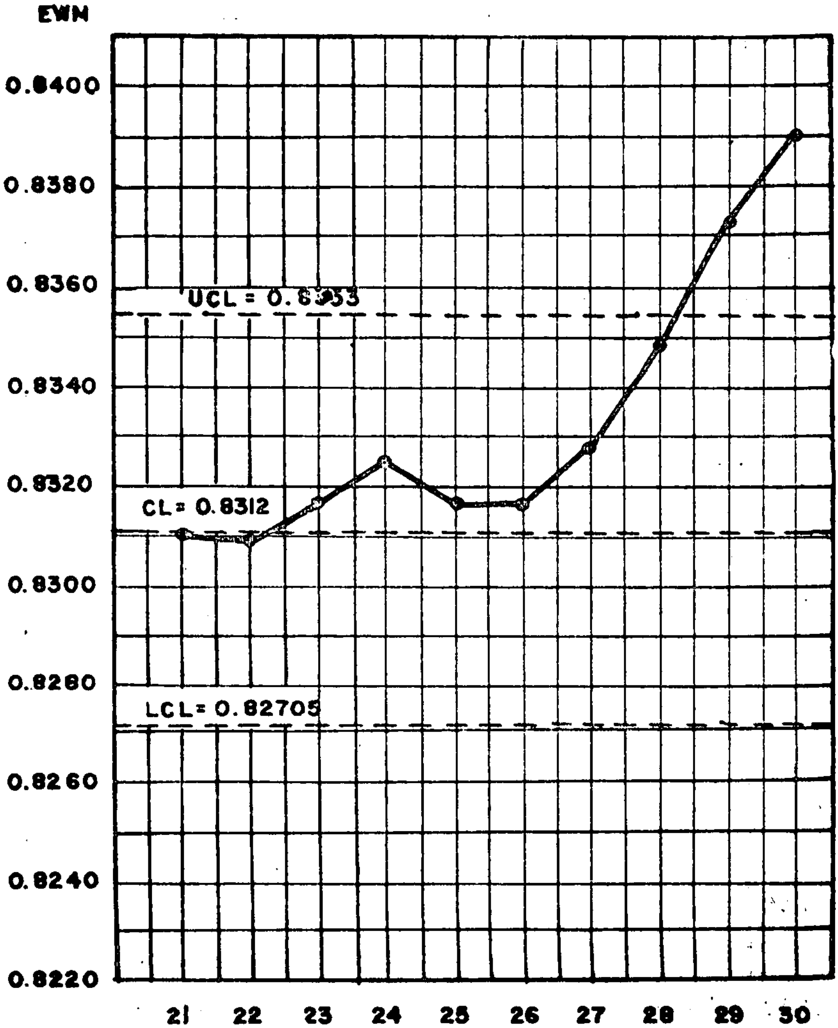


FIGURE 5. A CU-SUM CHART IN OPERATION